ONE CASE OF THE EXPLOSION OF A SYMMETRIC VARIABLE-THICKNESS SURFACE CHARGE

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Within the framework of the model of explosion phenomena proposed by M. A. Lavrent'ev, the plane problem of determining the shape of the excavation during the explosion of a constant-thickness surface charge is solved (when the pulse pressure is constant over the width of the charge). The problem of determining the shape of the excavation during the explosion of a surface charge whose section thickness varies linearly is considered below for the same model of explosion phenomena called "solid-liquid" [2]. (The problem reduces to a homogeneous linear Hilbert boundary-value problem with discontinuous coefficients. The solution is obtained in closed form, and recommendations are given for its practical realization.)

Let an excavation of cross section CDC_* (Fig. 1) be formed in the explosion of a long charge with section $ABB'A'B'_*B_*$ which is symmetric relative to the vertical axis. Let us investigate the case when there is a zone $MQNQ_*$ in the flow domain in which no motion originates; the boundary of this zone (we call it the rest zone) is the streamline along which the velocity equals the critical value. Because of symmetry, we examine just the right half of the flow domain which is denoted by G_z and its boundary which is denoted by $\Gamma_z (z = x + iy)$.

In conformity with the model taken for the explosion phenomenon, there exists a complex potential $w(z) = \varphi(x, y) + i\psi(x, y)$, where $\varphi = -\tilde{p}/\rho$, $\tilde{p}$ is the pressure pulse, $\rho = \text{const}$ is the density of the liquid (soil), and ψ is the stream function.

Let $AA' = q_1$ and $BB' = q_2$, where $q_1 < q_2$. Then the function characterizing the change in thickness of the charge is written as

$$\delta(x) = [(q_1 - q_2)/l]x + q_1 \quad (0 \leqslant x \leqslant l),$$

where $2l$ is the width of the charge. Taking into account that the pressure pulse is proportional to the thickness of the charge, i.e., $\tilde{p} = k\delta$ [3], where k is a known proportionality factor, we obtain the condition

$$\varphi = a_1 x + a_2 \quad (0 \leqslant x \leqslant l) \tag{1}$$

on the section AB of length l on the boundary Γ_z, where $a_1 = k(q_1 - q_2)/\rho l$; $a_2 = -kq_1/\rho$.

On the remaining sections of Γ_z we have

$$\psi = 0 \text{ on } AMQNDC, \quad \varphi = 0 \text{ on } BC, \\ v = v_0 \text{ on } MQN \text{ and } DC, \tag{2}$$

where $v_0 = \text{const}$ is a known critical velocity.

Therefore, according to conditions (1) and (2) either the real or the imaginary part of the function $w(z)$ which is analytic in G_z is known on the straight-line sections ABC, AM, and NHD of the boundary Γ_z, while $\operatorname{Im} w(z)$ and $|dw/dz|$ are known on the curvilinear sections MQN and DC. It is required to find the sections MQN and DC of the contour Γ_z (see Fig. 1).

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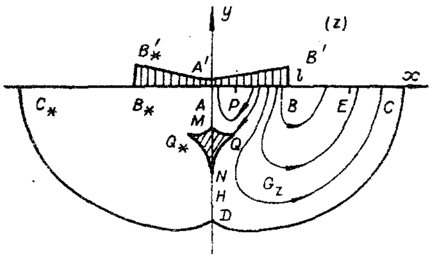

Fig. 1

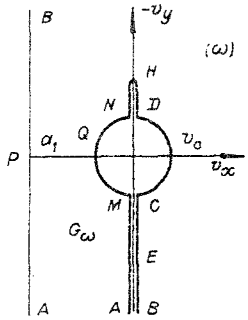

Fig. 2

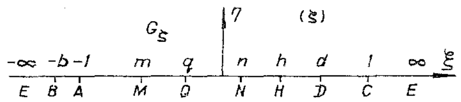

Fig. 3

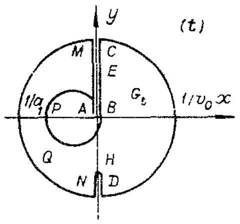

Fig. 4

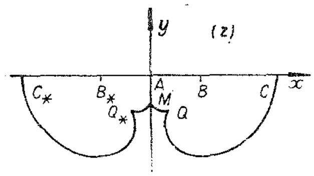

Fig. 5

Let us introduce the complex velocity hodograph plane

$$\omega = dw/dz = v_x + i(-v_y), \tag{3}$$

where v_x, v_y are projections of the velocity vector **v** on the coordinate axis. The representation

$$v_y = (2a_1/\pi)\ \ln|y| + F(y) \tag{4}$$

is valid for values of v_y on the section MA in the neighborhood of the point A, where F(y) is a function bounded in the neighborhood of the point A. Indeed, by mapping the domain G_z conformally on the half-plane Im $\zeta > 0$ the complex variable $\zeta = \xi + i\eta$ so that $\zeta = \infty$ would correspond to some internal point E of the section BC (see Fig. 1), and writing the Schwartz integral [4] for the function (3) which is analytic in the domain Im $\zeta > 0$, we obtain $v_x - iv_y = (a_1/\pi)\ln(a-\zeta) + f_1(\zeta)$, where $f_1(\zeta)$ is a function bounded in the neighborhood of the point $\zeta = a$ corresponding to the point A of the boundary Γ_z. Taking into account that the mapping function $z(\zeta)$ has the form $z(\zeta) = (\zeta - a)^{1/2} f_2(\zeta)$ in the neighborhood of the point a, where $f_2(\zeta)$ is bounded in the neighborhood of $\zeta = a$, $f_2(a) \neq 0$, we obtain the representation (4).

It is seen from (4) that $v_y \to +\infty$ as $y \to 0$ since $a_1 < 0$. Therefore, soil ejection into the atmosphere will occur in the neighborhood of the point A, to which the domain G_z shown in Fig. 1 corresponds.

The conditions

$$\begin{gathered} v_x = 0,\ v_y > 0 \text{ on } AM,\ CB; \\ v^2 = v_x^2 + v_y^2 = v_0^2 \text{ on } MQN,\ DC; \\ v_x = 0,\ v_y < 0 \text{ on } NHD; \\ v_x = a_1 < 0 \text{ on } BA. \end{gathered} \tag{5}$$

hold on separate sections of the boundary Γ_z.

Taking condition (5) into account, as well as the fact that we should have $v > v_0$ everywhere in the domain G_z, let us construct the domain G_ω in the ω plane (Fig. 2), which will correspond to the domain G_z (corresponding points in the different planes are denoted by the same letters).

Let us note that the location of the end of the slit H of the boundary Γ_ω of the domain G_ω is not known. At the same time, the coefficient a_2 of (1) characterizing the charge thickness is not used in the method proposed below for solving the problem. Hence, the location of the point H on Γ_ω, i.e., max $|v|$ on ND, will be given. Then the charge thickness and the shape of its corresponding excavation upon ejection of the soil will be determined during the solution of the problem.

Let the function $\omega = \omega(\zeta)$ map the half-plane Im $\zeta > 0$ of the variable $\zeta = \xi + i\eta$ conformally on the domain G_ω so that the points B, A, C of the boundary Γ_ω of the domain G_ω will correspond to the points b, −1, 1 ($1 < b < \infty$) of the ξ axis. Then some point E of the section BC on the boundary Γ_ω goes over into the infinitely remote point $\zeta = \infty$. The correspondence between the remaining points is shown in Fig. 3. The exact construction of the function $\omega(\zeta)$ which maps the circular heptagon G_ω conformally on a canonical domain is of great difficulty [5]. Hence, it is expedient to use one of the known approximate methods for the computations (see the remark).

Let us introduce the function $z(\zeta) = x(\xi, \eta) + iy(\xi, \eta)$ which is analytic in the half-plane Im $\zeta > 0$ and maps this half-plane conformally onto the domain G_z. Let us represent the function introduced by (3) as

$$\omega = |dw/dz| e^{-i\theta}, \tag{6}$$

where θ is the angle formed by the velocity vector $\mathbf{v}$ and the positive x axis. Since the sections MQ, QN, DC of the boundary Γ_z are streamlines, then $(\partial y/\partial\xi)/(\partial x/\partial\xi) = \tan\theta$ thereon, or

$$(\partial x/\partial\xi) \sin\theta - (\partial y/\partial\xi) \cos\theta = 0 \quad (m < \xi < n,\ d < \xi < 1).$$

where $x = x(\xi, 0)$; $y = y(\xi, 0)$. On the remaining sections of the ξ axis we have

$$\partial y/\partial\xi = 0 \text{ for } -\infty < \xi < -1,\ 1 < \xi < \infty;$$
$$\partial x/\partial\xi = 0 \text{ for } -1 < \xi < m,\ n < \xi < d.$$

Therefore, we have arrived at the problem of constructing a function $dz/d\zeta = \partial x/\partial\xi + i\,\partial y/\partial\xi$ which is analytic in the half-plane Im $\zeta > 0$ according to the boundary condition

$$c(\xi)\partial x/\partial\xi + d(\xi)\partial y/\partial\xi = 0, \tag{7}$$

where

$$c(\xi) = \sin\theta(\xi),\ d(\xi) = -\cos\theta(\xi) \text{ for } m < \xi < n,\ d < \xi < 1;$$
$$c(\xi) = 0,\ d(\xi) = 1 \text{ for } |\xi| > 1;$$
$$c(\xi) = 1,\ d(\xi) = 0 \text{ for } -1 < \xi < m,\ n < \xi < d.$$

Let us establish the class of functions in which the solution of (7) should be sought. For $\zeta = \zeta_A$ the function $z'(\zeta)$ has a half-power singularity. Here and henceforth, ζ_A, ζ_C, etc., denote values of ζ at the points A, C, etc.

It should be taken into account [4] in investigating the behavior of the function $z'(\zeta)$ in the neighborhood of the points M, Q, N, D, C that when the domain boundary in the neighborhood of an angular point consists of curvilinear sections which are analytic arcs, the behavior of the mapping function can differ substantially from its behavior in the case when the angular point is formed by straight-line sections.

Proceeding analogously to what has been done in [6], it can be shown that the function $z'(\zeta)$ has a singularity of order 1/2 at the point C, goes into a first order zero at the point Q and is bounded at the points D, N, M, and has a second-order zero at the point $\zeta = \infty$, $z'(\zeta)$.

Therefore, the solution of the linear homogeneous Hilbert boundary-value problem obtained (7) with discontinuous coefficients for the function $z'(\zeta)$ which is analytic in the half-plane Im $\zeta > 0$ must be sought in the class of functions which goes to zero in the first order at the point $\zeta = q$ and to zero in the second order at infinity, and is also bounded at the points m, n, d of the ξ axis while having half-order singularities at the point −1, 1 of the ξ axis.

Let us represent the function $z'(\zeta)$ as

$$dz/d\zeta = [(\zeta - q)/(\zeta + i)^3] g(\zeta), \tag{8}$$

where $g(\zeta) = \mu(\xi, \eta) + i\nu(\xi, \eta)$ is a function bounded at infinity and at the points m, q, n, d of the ξ axis, and has a half-order singularity at the points −1, 1 of the ξ axis. For $\eta = 0$ we have from (8)

$$\partial x/\partial\xi + i\,\partial y/\partial\xi = [(\xi - q)/(\xi + i)^3]\,[\mu(\xi) + i\nu(\xi)].$$

Hence, expressing $\partial x/\partial\xi$ and $\partial y/\partial\xi$, substituting in (7), and dividing out $(\xi - q)/(\xi^2 + 1)^3$, we obtain the boundary condition for $g(\zeta)$ as

$$a(\xi)\mu(\xi) + b(\xi)\nu(\xi) = 0, \tag{9}$$

where $a(\xi) = (\xi^3 - 3\xi)c(\xi) - (3\xi^2 - 1)d(\xi)$, $b(\xi) = (3\xi^2 - 1)c(\xi) + (\xi^3 - 3\xi)d(\xi)$. The Riemann problem corresponding to the Hilbert problem (9) is [7] $F^+(\xi) = G(\xi)F^-(\xi)$, where $F^+(\xi) = g(\xi)$, $G(\xi) = -[a(\xi) + ib(\xi)]/[a(\xi) - ib(\xi)]$, and the function $F(\zeta)$ satisfies the condition $\overline{F}^+(\zeta) = F^-(\zeta)$. On separate sections of the ξ axis we find

$$G(\xi) = [(\xi + i)/(\xi - i)]^3 \text{ for } |\xi| > 1, \quad G(\xi) = -[(\xi + i)/(\xi - i)]^3 \text{ for } \begin{cases} -1 < \xi < m, \\ n < \xi < d; \end{cases}$$

$$G(\xi) = [(\xi + i)/(\xi - i)]^3 e^{2i\theta(\xi)} \text{ for } \begin{cases} m < \xi < n, \\ d < \xi < 1. \end{cases}$$

In conformity with the class of solution established we take as the value of $\arg G(\xi) = \gamma(\xi)$

$$3\beta(\xi) + 2\theta(\xi) \text{ for } \xi_0 < \xi < 1; \; 3\beta(\xi) + 2\pi \text{ for } 1 < \xi < \infty;$$
$$3\beta(\xi) - 4\pi \text{ for } -\infty < \xi < -1; \; 3\beta(\xi) - 3\pi \text{ for } -1 < \xi < m;$$
$$3\beta(\xi) + 2\theta(\xi) \text{ for } m < \xi < n; \; 3\beta(\xi) - \pi \text{ for } n < \xi < d;$$
$$3\beta(\xi) + 2\theta(\xi) \text{ for } d < \xi < \xi_0,$$

where $d < \xi_0 < 1$; $\beta(\xi) = \arg[(\xi + i)/(\xi - i)]$ denotes the continuous branch for $-\infty < \xi < \infty$. The index of the Hilbert problem is

$$\varkappa = (1/2\pi)\,[\arg G(\xi_0 - 0) - \arg G(\xi_0 + 0)] = 0.$$

In this case the solution of the Hilbert problem is determined to the accuracy of a real constant [7, 8],

$$g(\zeta) = A_0\chi(\zeta), \quad A_0 = \overline{A}_0 = \text{const}, \tag{10}$$

where

$$\chi(\zeta) = \exp\left[\frac{1}{4\pi}\int_{-\infty}^{\infty}\left(\frac{\zeta + i}{\tau + i} + \frac{\zeta - i}{\tau - i}\right)\frac{\gamma(\tau)\,d\tau}{\tau - \zeta}\right].$$

The boundary value of $\chi(\xi)$ is expressed by the formula

$$\chi(\xi) = f(\xi)\exp i\gamma(\xi)/2, \tag{11}$$

where

$$f(\xi) = \exp\left[\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{1 + \xi\tau}{1 + \tau^2}\frac{\gamma(\tau)\,d\tau}{\tau - \xi}\right],$$

and the integral in the last expression is taken in the principal value sense.

On the basis of (8) and (10) we find

$$z(\zeta) = A_0\int_{-1}^{\zeta}\frac{\sigma - q}{(\sigma + i)^3}\chi(\sigma)\,d\sigma. \tag{12}$$

Performing the passage to the limit in (12) as $\zeta \to \xi$, and separating real and imaginary parts, we obtain the parametric equations of the boundary Γ_z:

$$\left.\begin{aligned} x &= A_0\int_{-1}^{\xi}(\sigma - q)f(\sigma)\Phi(\sigma)\,d\sigma, \\ y &= A_0\int_{-1}^{\xi}(\sigma - q)f(\sigma)\Phi_1(\sigma)\,d\sigma \end{aligned}\right\}(-\infty < \xi < \infty), \tag{13}$$

where

$$\Phi(\sigma) = [\sigma(\sigma^2 - 3)\cos(\gamma(\sigma)/2) + (3\sigma^2 - 1)\sin(\gamma(\sigma)/2)](\sigma^2 + 1)^{-3};$$

$$\Phi_1(\sigma) = [\sigma(\sigma^2 - 3)\sin(\gamma(\sigma)/2) - (3\sigma^2 - 1)\cos(\gamma(\sigma)/2)](\sigma^2 + 1)^{-3}.$$

Since only the condition of rectilinearity of the section NHD (Fig. 1) was taken into account during the solution, then by requiring that $\operatorname{Re} z_N = \operatorname{Re} z(n) = 0$, we obtain an equation to determine the parameter q from which

$$q = \int_m^n \sigma f(\sigma)\Phi(\sigma)\,d\sigma \Big/ \int_m^n f(\sigma)\Phi(\sigma)\,d\sigma. \tag{14}$$

The factor A_0 is found from the condition $z_B = z(-b) = l$:

$$l = -A_0 \int_{-b}^{-1} (\sigma - q) f(\sigma) \Phi(\sigma) \, d\sigma. \tag{15}$$

Since $dw/dz = \omega(\zeta)$, then taking account of (12)

$$w(\zeta) = A_0 \int_{1}^{\zeta} \frac{\sigma - q}{(\sigma + i)^3} \chi(\sigma) \omega(\sigma) \, d\sigma.$$

Hence, the value of the velocity potential φ at the point A, i.e., the quantity a_2 in (1), is written as

$$a_2 = -A_0 \operatorname{Re} \int_{-1}^{1} \frac{\sigma - q}{(\sigma + i)^3} \chi(\sigma) \omega(\sigma) \, d\sigma, \tag{16}$$

where the boundary value of $\chi(\xi)$ is determined from (11).

Knowing q and A_0, we find coordinates of points of the desired sections MQN and DC of the excavation boundaries from (13).

Therefore, by knowing the physical parameters of the medium ρ, v_0, the energy characteristic of the charge $\tilde{p}$ and by giving the quantities l, a_1 as well as the maximum velocity on the section ND (see Fig. 1), we find the quantity a_2 characterizing the thickness of the charge and the appropriate shape of the excavation upon ejection of the soil.

Remark. There is no need to construct all the functions $\omega(\zeta)$ mapping the domain G_ω onto Im $\zeta > 0$ in the method proposed to solve the problem. It is sufficient to find just its boundary values on the section $(-1, 1)$ of the ξ axis. To do this it is convenient to use electrical analog simulation [9]. First, the domain G_ω should be transformed into a domain bounded by the function $t = 1/\omega$, for example. The domain G_t obtained, which is bounded by an exterior circle of radius $1/v_0$ with center at the origin, and an interior circle of radius $1/2|a_1|$ with center shifted a quantity $1/2|a_1|$ on the real axis, as well as the slits NHD, AM, BC, is shown in Fig. 4. Applying one bus along the slit with the inner circle CBPAM and the other along the slit NHD, we map the domain G_t conformally on the rectangle MNDC (domain G_u with boundary Γ_u). Separating the bases MN and CD of the rectangle G_u into equal parts and taking into account (6) and the fact that $t = (1/v_0)e^{i\theta}$ on the boundary Γ_t of the domain G_t, we find a correspondence between points of the arcs MN, CD and the boundary Γ_t and points of the bases MN, CD of the rectangle G_u. Furthermore, applying one bus along the semicircle MQN and the other along the semicircle CD, we determine the position of the point B on the side MC of the rectangle G_u. We find the correspondence between points of the sections NHD and AM of the boundaries Γ_t and Γ_u if we take into account that we have $1/t = ve^{i\pi/2}$ and $1/t = ve^{3i\pi/2}$ on the sections NHD and AM of the boundary Γ_t. Then mapping the half-plane Im $\zeta > 0$ (see Fig. 3) onto the rectangle G_u by means of the Schwartz–Christoffel formula so that the points B, A, C of the boundary Γ_u would correspond to the points $-b$, -1, 1 of the ξ axis, we establish the dependence $\theta(\xi)$ on the sections (m, n), (d, 1) and the dependence $v(\xi)$ on the sections $(-1, m)$, (n, d) of the ξ axis. Knowledge of these dependences permits the determination of q by means of (14), A_0 by means of (15), the construction of the excavation by means of (13), and the determination of the quantity a_2 by means of (16).

Particular Case. Let us investigate the case when no rest zone originates in the excavation during explosion of the charge $ABB'A'B_*B_*$ (see Fig. 1). The excavation $CQMQ_*C_*$ corresponding to this case is shown in Fig. 5. Then the slit NHD will not be in the domain G_ω (see Fig. 2), i.e., $n = h = d$ on the ξ axis (see Fig. 3).

Now we will have in the boundary condition (7)

$$\begin{gathered} c(\xi) = \sin\theta(\xi),\ d(\xi) = -\cos\theta(\xi) \ \text{for} \ m < \xi < 1; \\ c(\xi) = 0,\ d(\xi) = 1 \ \text{for} \ |\xi| > 1; \\ c(\xi) = 1,\ d(\xi) = 0 \ \text{for} \ -1 < \xi < m. \end{gathered} \tag{17}$$

Taking account of (17) the form of the boundary condition (9) remains as before. We take as the value of $G(\xi) = \gamma(\xi)$

$$\begin{gathered} 3\beta(\xi) + 2\theta(\xi) \ \text{for} \ \xi_0 < \xi < 1;\ 3\beta(\xi) + 2\pi \ \text{for} \ 1 < \xi < \infty; \\ 3\beta(\xi) - 4\pi \ \text{for} \ -\infty < \xi < -1;\ 3\beta(\xi) - 3\pi \ \text{for} \ -1 < \xi < m; \\ 3\beta(\xi) + 2\theta(\xi) \ \text{for} \ m < \xi < \xi_0, \end{gathered}$$

where $m < \xi_0 < 1$ and $\beta(\xi)$ denotes the same as above. In this case the index of the Hilbert problem is also equal to zero. As before, the function $z(\zeta)$ is defined by (12).

However, now the possibility is provided of giving the charge shape entirely in advance, i.e., the quantity a_2. Then we will have the system of equations (15) and (16) to determine the constants A_0 and q. The initial data of the charge should hence satisfy the condition $\mathrm{Re}\, z(\xi) > 0$ for $q < \xi < 1$.

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HURLING OF SHELLS BY HOLLOW CHARGES

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Results of the numerical solution of the problem of one-dimensional hurling of shells by hollow explosive charges are elucidated. The results of the numerical solution are compared with asymptotic formulas. Numerous domestic and foreign papers have been devoted to the question of hurling shells by explosive charges. A numerical solution of the problem of convergence of a ring to the center under the effect of detonation products is presented in [1-3]. The problem of hurling a shell by a hollow explosive charge with an internal lining is considered in [4]; the solution of the problem of hurling a shell by a hollow explosive charge without the cavity lining is presented in [5] on the basis of the energy-balance equations; however, the complete picture of the processes occurring in the detonation products is not considered.

A shell with a hollow explosive charge is shown in Fig. 1. The detonation products (DP) are initially a gas at rest with the initial density $\rho_0 = \rho_{BB}$ and the pressure $p_0 = \rho_0 D^2/8$, whose extension is described by the Landau – Stanyukovich polytropy $p = A\rho^{\kappa}$ (k = 3).

The governing parameters of the problem are the load coefficient $\beta = m/M$ and the relative cavity radius $\lambda = a_{0p}/a_0$, where m is the mass of the high-explosive charge, M is the mass of the shell, a_{0p} is the radius of the cavity in the high-explosive charge, and a_0 is the inner radius of the shell. The shell strength and compressibility are neglected. The charge is in a vacuum.

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